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TECHNICAL NOTE

Dissipative cylindrical Couette flow of yield-pseudoplastic fluids

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INTRODUCTION

The problem of the laminar, steady, incompressible flow of Newtonian and non-Newtonian fluids between two infinitely long, rotating, concentric cylinders is one of great interest from a fundamental and practical point of view due to its application in many industrial processes, as well as in controlled shearing rate rheometry. In the latter case, the measurement of the rotational speed of the two cylinders and that of the torque applied to the cylinders (which is the same on both cylinders regardless of the fluid's rheology) allows for the analytical determination of the fluid's viscosity. In the case of a Newtonian fluid an analytic expression exists that relates the rotational speed to the torque [1], while in the case of non-Newtonian fluids analytic expressions exist for power-law [2] and Bingham [3] fluids. Some corrections for three-dimensional, i.e. end effects, can be easily incorporated into the governing equation through empirical constants.

The present work concentrates on the effects of viscous heating on the heat transfer problem associated with the flow of yield-pseudoplastic fluids between two rotating, infinitely long, concentric cylinders. Such a phenomenon is associated with rheological studies of highly viscous fluids in rotational rheometry. The rheological and thermophysical properties of many purely viscous, yield exhibiting fluids are very sensitive to variations in temperature. Therefore, the emerging temperature field due to viscous heating could complicate the characterization process and lead to significant errors in the interpretation of the obtained rheological measurements.

RESULTS

Details on the governing equations and the numerical method employed can be found in the report by Hammad *et al.* ref. [4]. The heat generated due to frictional losses is determined by the viscous dissipation term $\phi = \mu_{\text{eff}}^{*}\dot{\gamma}^{2}$. The radial distribution of this term determines the distribution of temperature in the flow field and consequently the Nusselt number distribution at the walls. Figure 1(a) shows the distribution of ϕ in the case of $r_i = 0.5$, for $Y = 1$ and two different power-law exponent values. The highest values of ϕ are experienced in the regions with the highest shear rates, i.e. at the surface of the inner, rotating cylinder. In addition, for the lower value of n a plug zone appears in the vicinity of the outer stationary wall, because the yield stress exceeds the shear stress values in the zone. In this case, the value of ϕ at the inner wall exceeds the one corresponding to the higher n value, and as a result the two curves cross each other. Given that within the plug zone there is no deformation of the fluid elements the viscous dissipation term becomes $\phi = 0$. At a smaller gap of $r_i = 0.9$ (Fig. 1(b)), the value of ϕ increases overall, due to the increased levels of shear rates. In this case, for both values of the power-law exponent the flow is fully yielded throughout the domain. A lower power-law exponent still results in lower overall stress levels and higher rates of deformation at the inner cylinder boundary. However, contrary to the case where unyielded zones exist, in this case the combined result of the drop in stress levels and increase of rates of deformation is the overall decrease in the values of ϕ across the gap width. As the yield number increases, see Fig. 1(c)–(d), the plug zone appears both for $n = 1$ and $n = 0.4$. Again, the plug zones are characterized by zero values of the dissipation function. Also, the characteristic crossover of the curves appears as the power-law exponent is decreased.

Due to the fact that the value of the dissipation function reaches its maximum value at the inner wall and the fact that the inner wall is adiabatic, the maximum temperature within the fluid also appears at the inner wall surface. Figure 2(a)–(c) shows the variation of this maximum temperature with the yield number and power-law exponent for the three different gap widths of $r_i = 0.5$, 0.9 and 0.99. For a large gap width of $r_i = 0.5$, this temperature is rather insensitive to the power-law exponent and depends mostly on the yield number. The dependence on the yield number is also weak at low values and becomes significant only once the yield number exceeds the level that corresponds to the appearance of a plug zone. Once the plug zone appears, the thermal resistance at the outer surface increases since heat transfer through this zone is due to conduction only. Given that the heat generated within the fluid has to be transferred out of the system for steady conditions to exist, an increased temperature difference needs to be established. As a result the maximum temperature increases with the yield number and the corresponding increase of the plug/conduction-only zone. As the gap width decreases, the appearance of the plug zone is displaced to higher yield number and power-law exponent values. Therefore, for fixed yield and power-law exponent values the maximum temperature decreases as the gap width decreases. This is demonstrated in Fig. 2(b)–(c). Notice that in this case the dependence on n is rather significant at lower yield number values, while it is again insignificant as the yield number reaches very high values. The crossover of the curves at the higher end of the yield number spectrum is due to the earlier establishment of the plug zone at lower n values, which is associated with the rapid increase in the thermal resistance within the plug.

The Nusselt number distributions along the stationary outer cylinder, which is assumed to be at a constant temperature of $\theta = 0$, is shown in Fig. 3(a)–(c) again for the

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NOMENCLATURE

$\bar{k}$	thermal conductivity of fluid
K	consistency index
n	power-law exponent
Nu	Nusselt number $d\theta/dr/\theta_b$
r	non-dimensional radial distance, R/R_o
r_i	non-dimensional inner wall distance R_i/R_o
r^+	normalised non-dimensional radial distance $(r-r_i)/(1-r_i)$
R_i	radius of the inner cylinder
R_o	radius of the outer cylinder
T	temperature
Y	yield number, $\tau_y/K\Omega_c^n$.

Greek symbols

$\dot{\gamma}$	shear rate
θ	non-dimensional temperature $(T-T_c)/K\Omega_c^{n-1}(R_o^2\Omega_c^2/\bar{k})$
μ_{eff}^*	non-dimensional effective viscosity, $\mu_{eff}/K\Omega_c^{n-1}$
μ_{eff}	effective viscosity, $K\{Rd\Omega/dR\}^{n-1}+\tau_y/R\,(d\Omega/dR)$
τ_y	yield stress
ϕ	viscous dissipation term, $\mu_{eff}^*\dot{\gamma}^2$.

Subscripts

b	bulk properties
i	inner wall
o	outer wall.

three radii ratios of $r_i = 0.5$, 0.9 and 0.99. For the wider gap case, Fig. 3(a), and at higher yield numbers the Nusselt number attains low values which are practically independent of the power-law exponent. The flow field in this region is characterized by rather small velocities due to the tendency for the formation of a plug flow region, thus conductive heat transfer becomes the dominant factor resulting in the small Nusselt numbers. At lower yield numbers, higher power-law exponents result in higher Nu due to the fact that the amount of heat generated increases with the exponent and the slope of the velocity profile at the outer wall increases to accommodate the increased bulk flow rate. As the gap width decreases, the shearing rates at the wall increase, resulting in much more effective convective heat transfer. Consequently, as the radii ratio increases from 0.5 to 0.9 and 0.99, respectively (see Fig. 3(b)–(c)), Nusselt numbers that are one and two orders of magnitude higher are obtained. At the higher yield number range, the Nusselt numbers drop significantly, again due to the tendency for the formation of the plug flow zones which are always characterized by very high thermal resistances and low Nusselt numbers.

The implications of the above results on the characterization of yield-pseudoplastic fluids via rotational rheometry can be rather profound. As an example, consider the characterization of an industrial grease such as the ones analyzed by Cho *et al.* [5]. A typical grease has a yield stress of $\tau_y = 900$ Pa, a consistency index of $K = 165$ Pa s^n and a power-law exponent of $n = 0.2$. A typical cylindrical rotational rheometer (Brookfield, LDV-III) has a 25.4 mm diameter fixed outer cylinder and a 12.7 mm diameter rotating inner one. The rotational speeds vary between 0.00167 and 4.17 rev s^{-1} (0.1 to 250 rpm). Considering a speed of 1.67 rev s^{-1} (100 rpm), the yield number is calculated to be $Y = 0.40$. From Fig. 2(a), the corresponding non-dimensional maximum temperature rise is about $\theta_i = 0.3$. The corresponding temperature rise is then calculated to be about 6°C. As is shown in ref. [5], such a temperature rise results in a decrease of 50% in the effective viscosity of the grease! It is clear therefore that neglect of the viscous heating effects can lead to erroneous results for the viscosity.

CONCLUSIONS

The non-isothermal flow of a yield-pseudoplastic fluid between two concentric cylinders of which the inner one is rotating at a constant speed, while the outer one is stationary has been studied. The temperature field is generated by the viscous dissipation of the mechanical energy of the fluid flow. The results demonstrate the strong effects that the radius ratio of the two cylinders and the rheological properties of the fluid (yield stress and power-law exponent) have on the thermal field and the Nusselt number at the boundary. The yield number has a profound effect on the temperature field and higher values of this parameter are associated with decreasing heat transfer rates and increased bulk temperatures. Diminishing gap widths result in increased Nusselt numbers. As a result, large gaps in combination with high yield numbers can seriously compromise the accuracy of rheological characterization of such fluids via rotational rheometry. The implications of the above results for rotational rheometry are rather significant. High yield number fluids generate large amounts of heat which in turn increase the operating temperature. Typically, the rheological properties are sensitive to temperature. As a result, the temperature gradients within the fluid can strongly affect the measurements and greatly compromise their accuracy.

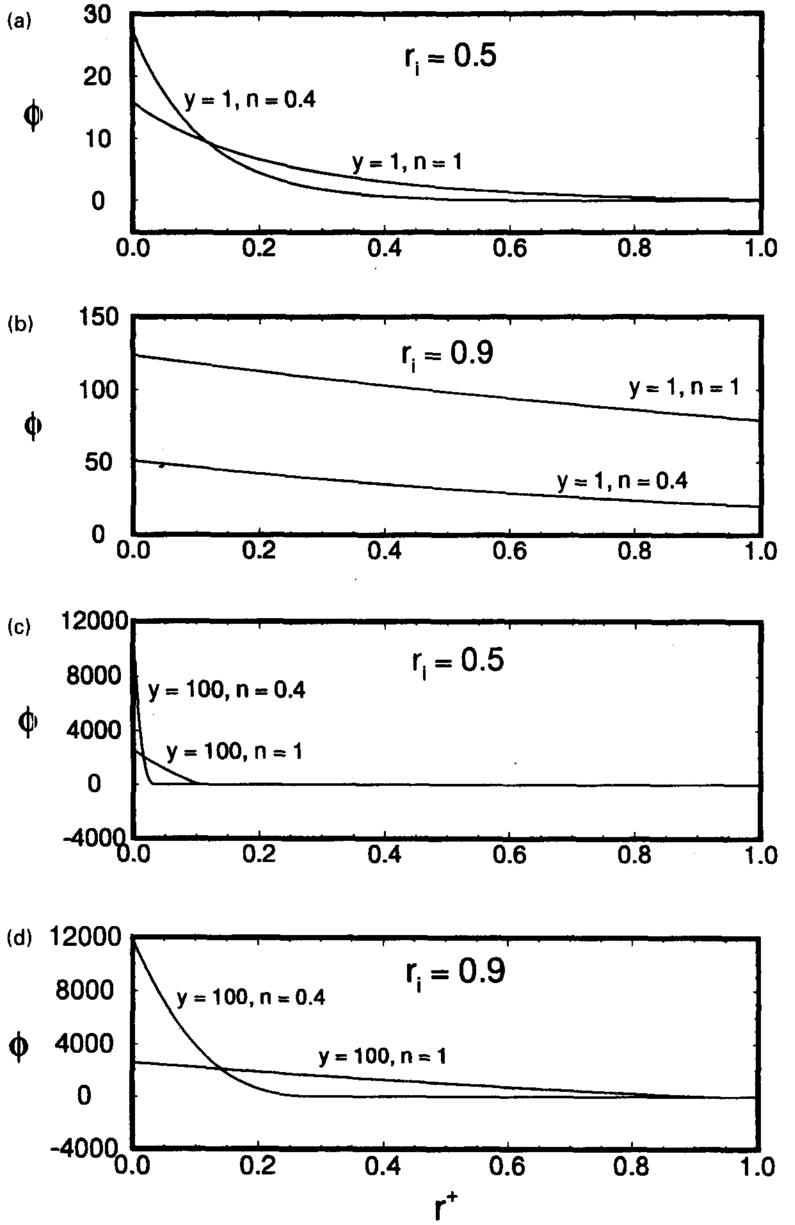

Fig. 1. Heat generation term distribution in the fluid for (a) $Y = 1$ and $r_i = 0.5$; (b) $Y = 1$ and $r_i = 0.9$; (c) $Y = 100$ and $r_i = 0.5$; (d) $Y = 100$ and $r_i = 0.9$.

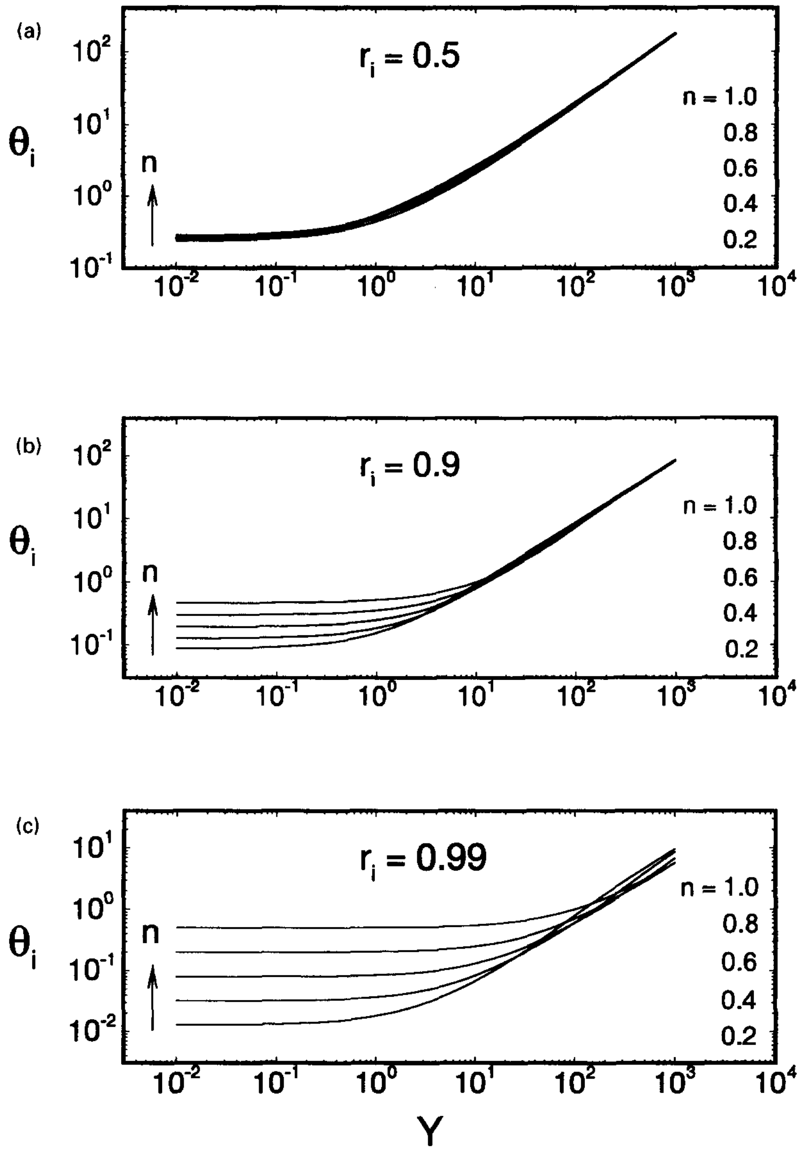

Fig. 2. Maximum fluid temperature variation with the yield number and the power-law exponent for (a) $r_i = 0.5$; (b) $r_i = 0.9$; (c) $r_i = 0.99$.

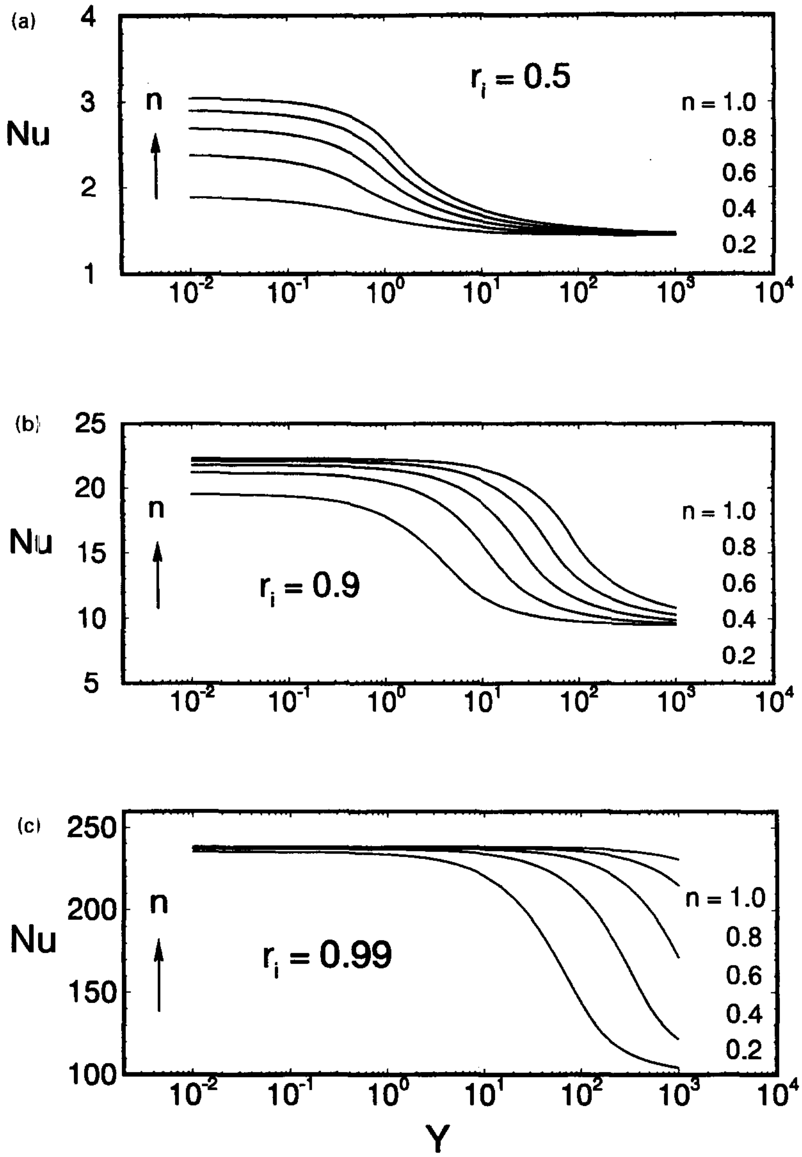

Fig. 3. Nusselt number variation with the yield number and the power-law exponent for (a) $r_i = 0.5$; (b) $r = 0.9$; (c) $r_i = 0.99$.

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